Grand unification in higher dimensions with split supersymmetry

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ABSTRACT: We investigate gauge coupling unification in higher dimensional GUT models with split supersymmetry. We focus on 5d and 6d orbifold GUTs, which permit a simple solution to several problems of 4D GUTs as well as control over GUT scale threshold corrections. In orbifold GUTs, calculable threshold corrections can raise or lower the prediction for $\alpha_s(M_Z)$ in a way that depends on the location of Higgs fields. On the other hand, split supersymmetry lowers the prediction for $\alpha_s(M_Z)$. Consequently, split supersymmetry changes the preferred location of the Higgs fields in orbifold GUTs. In the simplest models, we find that gauge coupling unification favors higgs doublets that live on the orbifold fixed points instead of in the bulk. In addition, relatively high scales of supersymmetry breaking of $10^{10\pm 2}$ GeV are generically favored.

KEYWORDS: GUT, Supersymmetric Standard Model, Beyond Standard Model.

Contents

1. Introduction

Weak scale supersymmetry provides a nice solution to the naturalness problem and a predictive framework for electroweak symmetry breaking. It also successfully predicts gauge couplings unification at a scale of $\approx 2 \times 10^{16}$ GeV thereby providing support for the idea that the standard model is embedded in a grand unified theory (GUT) [1]. Recently, Arkani-Hamed and Dimopoulos proposed that naturalness may not be a good criterion for determining weak scale physics [2]. Instead, they proposed that the higgs mass is fine-tuned and looked for alternative motivations for supersymmetry not tied to naturalness. Giudice and Romanino investigated this possibility further and found that the MSSM with heavy scalar superpartners and light fermionic superpartners emerged naturally by demanding gauge coupling unification and a viable dark matter candidate [3]. This framework is known as split supersymmetry (split-SUSY) and further model building and phenomenological consequences have been worked out in [4].

In this paper, we focus on the issue of gauge coupling unification and the embedding of split-SUSY in a viable GUT model. We focus on orbifold GUT models with a single extra dimension because these provide a particularly nice solution to many of the standard problems with 4d GUTs and a calculable framework for high-scale threshold corrections [5–9]. In the simplest 4d supersymmetric SU(5) GUTs, the prediction for the strong coupling constant at the scale M_Z is $\alpha_s^{MSSM,GUT}(M_Z) = 0.130 \pm 0.004^1$, somewhat larger than the experimentally measured value of $\alpha_s^{exp} = 0.119 \pm 0.002$ [11]. One of the important aspects of split-SUSY is a prediction for $\alpha_s(M_Z)$ that is smaller than in 4d GUT scenarios. Likewise,

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[1]We've neglected including threshold contributions in quoting this prediction, [10].

one of the important features of orbifold GUTs is the presence of threshold corrections to gauge couplings coming from heavy KK states that can improve the agreement between the predicted value of $\alpha_s(M_Z)$ and experiment. The primary goal of this paper is to show how the competing effects on $\alpha_s(M_Z)$ from split-SUSY and 5d orbifold GUT thresholds constrain the structure of generic orbifold GUTs. Contrary to the findings of previous authors [9, 12 – 14] that bulk higgs fields are preferred for gauge coupling unification in low-energy SUSY models, we find that brane localized higgs fields are naturally preferred in the split-SUSY scenario.

This result is attractive for several reasons. First, if the Higgs fields live on a brane, then the simplest possibility of just having the MSSM or SM Higgs doublets can be realized [15]. Moreover, brane higgs scenarios are readily compatible with inherently 4d mechanisms of electro-weak symmetry breaking. Of course, we do lose a few nice features with brane Higgs doublets such as unified quark-lepton mass relations and charge quantization, but in any case SU(5) quark-lepton mass relations for the first two generations are seemingly inconsistent with experiment.

Our paper is outlined as follows. In section 2, we'll review the framework of orbifold GUTs with extra dimensions. Although the findings of our analysis will apply quite generally, we'll focus on the class of models developed in [9, 12 – 14] to illustrate the important features of orbifold GUTs with split-SUSY. In section 3, we'll discuss the effects of raising the SUSY breaking scale above M_Z as well as the competing effects of heavy KK state thresholds. In section 4, we'll present the results of our two-loop analysis of gauge coupling unification and our findings for the favored range of the SUSY breaking scale and the location of the higgs fields in an orbifold GUT completion. We then end with concluding remarks.

2. GUTs with extra dimensions

Among the many successes of 4d SUSY GUTs is the explanation for charge quantization and the pattern of quark and lepton quantum number, a prediction for gauge coupling unification close to experimental bounds, quark-lepton mass relations that reduce the number of flavor parameters in the standard model, and a robust framework for generating small nonzero Majorana neutrino masses. However, important issues remain unresolved in these models. Chief among these is the predicted rate of proton decay from colored higgsino exchange that essentially excludes the simplest SU(5) GUTs. Other problems include the origin of SU(5) breaking, the very large splitting in mass required between the Higgs colored triplet states and weak doublets, the fact that the observed quark/lepton mass relations for the lighter generations violate generic GUT relations, and little explanation for the other flavor hierarchies of the standard model.

GUT models with extra dimensions offer several possible resolutions to these problems. As in [9, 12 – 14], one can construct a 5d or higher dimensional model with SU(5) gauge symmetry and break the symmetry down to SU(3) × SU(2)$_L$ × U(1)$_Y$ on 4d branes using boundary conditions. For example, in 5d orbifold theories, orbifold fixed points serve as the branes on which the SM fields can live. The boundary conditions emerge as a consequence

of requiring the fields to transform with definite parity under the orbifold group. With such a setup, the quark and lepton families and higgs of the standard model can be added to either the bulk or to the branes.

The low energy effective theory consists of the lightest states in the KK expansion of fields. What should we require of the low energy theory? In our case, we want to recover the MSSM at low energy. All of the SU(5) triplet states should be heavy, and proton decay should be suppressed to experimentally acceptable levels. To see how these requirements can be naturally satisfied in a higher dimensional GUT, let's consider the specific model developed in [12]. Not only does this model illustrate the important features of higher dimensional GUTs, but we will use this model to concretely study gauge coupling unification with split-SUSY later.

2.1 A 5d orbifold GUT model

Based on [12], the model we'll consider contains a single extra dimension. The orbifold we consider is obtained by taking the real line and identifying under translations by $2\pi R$ and the discrete reflection $Z : y \to -y$. This is just $\frac{S^1}{Z}$ with two orbifold fixed points (orbifold branes) located at $y = 0$ and $y = \pi R$ where R is the radius of S^1 [12]. It is assumed that the bulk theory has 5d N=1 SUSY so that there is a natural way to obtain 4d N=1 SUSY on the orbifold branes. SU(5) gauge fields are taken to reside in the bulk so that boundary conditions can break the symmetry down to $SU(3) \times SU(2)_L \times U(1)_Y$ on at least one of the two orbifold branes. Thus, bulk gauge fields reside in a 5d N=1 SUSY vector multiplet, (V, Σ), consisting of a 4d N=1 SUSY vector multiplet V and chiral multiplet Σ transforming in the adjoint of SU(5). Matter fields can reside either in the bulk or on the branes. Bulk matter fields reside in 5d N=1 SUSY hypermultiplets, (Φ, Φ^c), consisting of 4d N=1 SUSY chiral and anti-chiral multiplets Φ and Φ^c respectively.

Under the orbifold actions, the constituent multiplets transform with definite parity. To preserve a single N=1 SUSY in the zero mode spectrum, the gauge fields V and Σ are given Z parities of $+$ and $-$ respectively. Bulk hypermultiplet constituents Φ and Φ^c are given parities $+$ and $-$ respectively. Translations by $2\pi R$ act on the fundamental of SU(5) as $P = (+, +, +, -, -)$, and so break SU(5). In addition, there can be extra factors of $\eta_\Phi = \pm 1$ for bulk hypermultiplets. The above parity assignments lead to boundary conditions on the fields at the orbifold fixed points,

$$
\begin{aligned}
V^{\pm}(x^\mu, y) &= V^{\pm}(x^\mu, -y) = \pm V^{\pm}(x^\mu, y + 2\pi R), \\
\Sigma^{\pm}(x^\mu, y) &= -\Sigma^{\pm}(x^\mu, -y) = \pm \Sigma^{\pm}(x^\mu, y + 2\pi R), \\
\Phi^{\pm}(x^\mu, y) &= \Phi^{\pm}(x^\mu, -y) = \pm \eta_\Phi \Phi^{\pm}(x^\mu, y + 2\pi R), \\
\Phi^{c\pm}(x^\mu, y) &= -\Phi^{c\pm}(x^\mu, -y) = \pm \eta_\Phi \Phi^{c\pm}(x^\mu, y + 2\pi R),
\end{aligned}
\tag{2.1}
$$

where the superscript $\pm$ refers to the parity under the SU(5) breaking action of translations. At $y = 0$, we see that bulk 5d N=1 (4d N=2) SUSY has been broken down to 4d N=1 by Z, but SU(5) is still operative. At $y = \pi R$, the bulk SUSY has been broken down to 4d N=1 and SU(5) has been broken with only $SU \times SU(2)_L \times U(1)_Y$ surviving.

A single generation of standard model fermions can live in the bulk if they reside in two hypermultiplets transforming in the **10**, $T(u, e)$ and $T'(q)$, and two in the **$\bar{5}$**, $F(d)$ and $F'(l)$. With the choice $\eta_T = \eta_F = 1$ and $\eta_{T'} = \eta_{F'} = -1$, the zero modes of these four hypermultiplets fill out a single generation of standard model fermions. There is also the possibility of part of a generation living on an orbifold brane with the remainder in the bulk. For example, d and l can come from a 4d N=1 susy **$\bar{5}$** multiplet living on a brane while the u, e, and q components come from two **10**s in the bulk.

If the Higgs fields arise from the bulk, they can come from two bulk hypermultiplets transforming in the **5** and **$\bar{5}$** [12]. Another possibility is for the Higgs to arise from a vector multiplet. For example, Higgs fields with the correct quantum numbers can come from the doublet components of the adjoint of SU(6) under its $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ decomposition. 6d orbifold models with this feature have been constructed in [16]. Of course, the Higgs can also be a brane field [15].

Having introduced the orbifold GUT framework, we can see how the problems of standard 4d SUSY GUTs can be resolved. The Higgs doublet-triplet splitting problem is solved by requiring boundary conditions that eliminate the triplet zero mode. The Higgs triplet states are now naturally heavy with mass of order, $1/M_c$, where $M_c = 1/R$ is the scale of the extra dimension. We also have the possibility of placing Higgs fields on the 4d SU(5) violating brane in which case there are not necessarily any triplet partners to begin with.

Yukawa couplings of bulk hypermultiplets are forbidden by 5d supersymmetry and so the yukawa couplings reside on the branes. Thus, if the Higgs is in the bulk, then proton decay from dimension five triplet higgsino exchange via dirac mass terms is eliminated by the bulk supersymmetry. Additional dangerous sources of proton decay can be eliminated by using the bulk $\mathrm{SU}(2)_R$ symmetry that comes from the N=2 4d SUSY of the 5d bulk [12].

SU(5) mass relations can be preserved for the heaviest generation by placing it on the SU(5) brane at $y = 0$. If the first two generations are placed in the bulk, then their masses will not respect SU(5) relations because the down-type quarks and charged leptons have different yukawa couplings to the Higgs field for our choice of representation. Moreover, because the bulk fields are spread out in the extra dimension, wave-function suppression will naturally make their masses smaller thereby explaining why heavy matter fields satisfy SU(5) mass relations while light matter does not.

Gauge coupling unification can proceed as usual except now there will be radiative corrections coming from KK modes and brane localized gauge kinetic operators that do not respect the bulk SU(5). As long as the extra dimension is large compared to the unification scale, then the bulk gauge kinetic operators will dominate over brane localized operators by a factor of $\frac{M_s}{M_c}$. Of course, this assumes that we can reliably estimate the couplings to be of comparable strength at some scale. We will later identify the unification scale with the scale of strong coupling for the 5d theory thereby justifying this assumption.

In this paper, we will also briefly consider 6d SO(10) orbifold GUTs. SO(10) orbifold GUTs on $\frac{T^2}{Z_2}$ have been constructed and their features discussed in the literature [17]. Other 6d orbifold GUTs with unified gauge group SU(6) have been constructed that contain Higgs doublets arising from bulk gauge fields [16]. For our purposes, the primary impact on

gauge coupling unification that 6d models introduce consists of different power law scaling of couplings above the scale M_c than in 5d models. As will be made explicit in section 3, the primary effect of a sixth dimension will be to decrease the overall magnitude of the contribution to α_s from KK thresholds.

3. Gauge coupling unification in split-SUSY

In this section, we discuss gauge coupling unification in split-SUSY. We start by considering the experimentally measured values, $\sin^2 \theta_W(M_Z) = 0.23150 \pm 0.00016$, $\alpha^{-1}(M_Z)) = 128.936 \pm 0.0049$, and $\alpha_s(M_Z) = 0.119 \pm 0.003$ [11]. Given $\sin^2 \theta_W(M_Z)$ and $\alpha^{-1}(M_Z))$, we can obtain a prediction for $\alpha_s(M_Z)$ assuming unification at a high scale. A one-loop analysis of this prediction will give a prediction for $\alpha_s(M_Z)$ with errors dominated by the large SU(3) coupling of order $(\alpha_s^{1-\text{loop}}(M_Z))^2$. However, the experimental uncertainty is of order $.003 \approx (\alpha_s(M_Z))^3$, so a full two-loop analysis with one-loop thresholds is needed to reliably compare theory with experiment.

The easiest way to calculate gauge coupling predictions is to use a succession of effective field theories (EFTs) obtained by integrating out heavy particles at the appropriate mass scales [18]. In this way, we can use a simple mass independent renormalization scheme such as $\bar{M}S$ (or $\bar{D}R$ if we're working with SUSY) in each effective theory. The effects of large log contributions at lower energies coming from massive states is absorbed into the matching conditions between the theories. Assuming that the gauge couplings are unified at a scale M_G, the evolution of the coupling down to M_Z proceeds in two steps. In the first step, the tower of KK modes contribute at one-loop to the gauge couplings above the compactification scale M_c. From the 5d perspective, the theory is not renormalizable which is reflected by the mass dimension of the 5d gauge coupling. Thus, we expect power-law scaling of the gauge couplings between the GUT scale M_G and M_c. In the second step, the gauge couplings run in the usual logarithmic fashion from M_c to M_Z.

For clarity, we start by discussing the familiar logarithmic running and matching below the scale M_c. After studying the one-loop effects of lifting the SUSY breaking scale m_S, we will discuss the scaling of the couplings above M_c.

3.1 Running and matching gauge couplings

The underlying UV theory is the model presented in section 2. After matching between the 5d theory and the effective 4d theory at the compactification scale M_c, we obtain the MSSM. In the split-SUSY scenario, we assume that the squarks, sleptons, charged and pseudoscalar Higgs are degenerate with mass m_S. Below the scale m_S, the effective theory consists of only the higgsinos $\tilde{H}_{u,d}$, gluinos $\tilde{g}^\alpha$, W-inos $\tilde{W}^a$, B-ino $\tilde{B}$, and the standard model fields with a single higgs doublet H,

$$\mathcal{L}_{SSSM} = \mathcal{L}_{gauge} + m^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 -$$
$$-[y_{ij}^u \bar{q}_j u_i(i\sigma_2 H^*) + y_{ij}^d \bar{q}_j d_i H + y_{ij}^e \bar{l}_j e_i H +$$
$$+\frac{M_3}{2}\tilde{g}^\alpha \tilde{g}^\alpha + \frac{M_2}{2}\tilde{W}^a \tilde{W}^a + \frac{M_1}{2}\tilde{B}\tilde{B} + \mu \tilde{H}_u^T(i\sigma_2 \tilde{H}_d) +$$
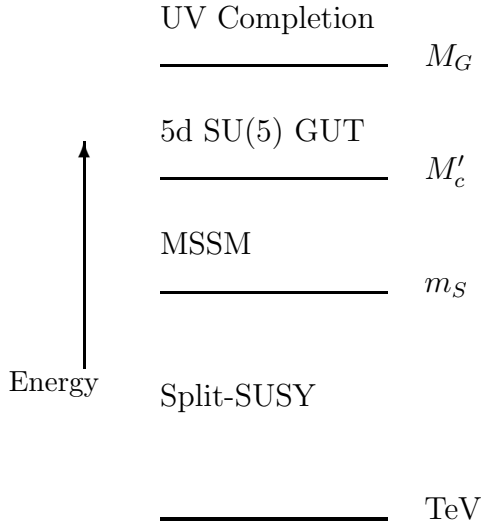
Figure 1: Summary of the effective theories and their relevant scales in a 5d SU(5) orbifold GUT with split-SUSY

$$+\frac{1}{\sqrt{2}}H^\dagger(\tilde{g}_u\sigma^a W^a + \tilde{g}'_u\tilde{B})\tilde{H}_u + \frac{1}{\sqrt{2}}(H^T i\sigma_2)(-\tilde{g}_d\sigma^a\tilde{W}^a + \tilde{g}'_d\tilde{B})H_d] + \text{h.c.}$$

With H_u and H_d the up-type and down-type MSSM Higgs doublets respectively, we fine tune the linear combination $H = -\cos(\beta)i\sigma_2 H_d^* + \sin(\beta)H_u$ to be light. The tree level matching condition between the MSSM and the split-susy lagrangian can be found by taking $H_u \rightarrow \sin(\beta)H$ and $H_d \rightarrow \cos(\beta)i\sigma_2 H^*$ in the MSSM lagrangian, so that

$$\begin{aligned}
\lambda(m_S) &= \frac{g_2^2 + \frac{3}{5}g_1^2}{4}\cos^2(2\beta), \\
y_{ij}^u &= \lambda_{ij}^u \sin(\beta), \\
y_{ij}^{d,e} &= \lambda_{ij}^{d,e}\cos(\beta), \\
\tilde{g}_u &= g_2\sin(\beta), \\
\tilde{g}_d &= g_2\cos(\beta), \\
\tilde{g}'_u &= \sqrt{\frac{3}{5}}g_1\sin(\beta), \\
\tilde{g}'_d &= \sqrt{\frac{3}{5}}g_1\cos(\beta),
\end{aligned} \tag{3.1}$$

where $\lambda_{ij}^{u,d,e}$ are the Higgs yukawa coupling matrices in the MSSM. To obtain predictions for standard model couplings at the weak scale M_Z, we also need to match the split-SUSY theory onto a low energy theory containing just the standard model fields [19].

With our collection of effective theories (illustrated in figure 1), we RG run the MSSM from the compactification scale M_c to the scale of supersymmetry breaking m_S [3]. We then match onto the split-SUSY effective lagrangian (3.1) using the matching conditions (3.1). After RG running the split-susy couplings to the scale of the higgsino and gaugino masses,

we match onto the standard model. Finally, we match the low-energy parameters to experimentally measured couplings and extract a prediction for $\alpha_s(M_Z)$ using measured values of $\sin^2\theta_W(M_Z)$, $\alpha(M_Z)$, and masses of the standard model fermions as inputs.

In matching the theories above and below m_S, we must consider several large threshold corrections. First, the running of the gaugino and higgsino masses can lead to a significant spread in masses at low energy, so separate thresholds are included for each. The heavy top mass also contributes a large threshold above the weak scale, so it is also included.[2] We should in principle include all of the one-loop thresholds coming from the sparticle and higgs scalar spectrum at m_S, but it is a decent approximation to ignore these effects given that our theory is weakly coupled at m_S for m_S sufficiently large.

3.2 One-loop analysis

Using the RGEs in [3], we can obtain some insight into the consequences of split-SUSY and GUT thresholds for gauge coupling unification by calculating their one-loop effects. First, we focus on the effects of lifting m_S to a high scale and so we momentarily ignore GUT thresholds. The gauge couplings $\alpha_i(M_Z)$ can be written as

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(M_G)} + \frac{b_i^{MSSM}}{2\pi}\log\frac{M_G}{M_Z} +$$
$$+ \frac{b_i^{SSSM} - b_i^{MSSM}}{2\pi}\log\frac{m_S}{M_Z} + \frac{b_i^{SM} - b_i^{SSSM}}{2\pi}\log\frac{M}{M_Z} + \gamma_i + \delta_i, \quad (3.2)$$

where M_G is some high mass scale (either the compactification scale, or if we were doing conventional unification the GUT scale), M is the mass scale of gauginos and higgsinos, and $b_i^{SM} = (\frac{41}{10}, -\frac{19}{6}, -7)$, $b_i^{MSSM} = (\frac{33}{5}, 1, -3)$, and $b_i^{SSSM} = (\frac{9}{2}, -\frac{7}{6}, -5)$ are the β-function coefficients for the standard model, the MSSM, and split-SUSY respectively. The two-loop contributions are contained in the γ_i factors and additional small threshold factors are included in δ_i. In the limit $m_S = M = M_Z$, an easy way to obtain a prediction for $\alpha_s(M_Z) = \alpha_3(M_Z)$ is to take the linear combination $\alpha_3^{-1} - \frac{12}{7}\alpha_2^{-1} + \frac{5}{7}\alpha_1^{-1}$. Using (3.2), the $\log\frac{M_G}{M_Z}$ term cancels out and we are left with

$$\alpha_3^{-1}(M_Z) = \frac{12}{7}\alpha_2^{-1}(M_Z) - \frac{5}{7}\alpha_1^{-1}(M_Z) + \gamma_3 - \frac{12}{7}\gamma_2 + \frac{5}{7}\gamma_1 + \delta_3 - \frac{12}{7}\delta_2 + \frac{5}{7}\delta_1, \quad (3.3)$$

where we've assumed that $\alpha_i(M_G) \approx \alpha(M_G)$. Feeding in the experimentally measured values of $\alpha_{1,2}(M_Z)$, one obtains $\alpha_s^{MSSM,GUT}(M_Z) = 0.130 \pm 0.004$ for the MSSM [10].

When $m_S, M \geq M_Z$, we can take the same linear combination as in (3.3) to calculate $\alpha_s(M_Z)$. The difference $\delta\alpha_s$ between $\alpha_s^{MSSM,GUT}(M_Z)$ and the value calculated in the split-SUSY case is then approximately,

$$\delta\alpha_s(M_Z) \approx -\frac{\alpha_s(M_Z)^2}{2\pi}\Delta, \quad (3.4)$$

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[2]The difference between the $\bar{MS}$ running and pole mass can lead to additional threshold correction that should also be included. In our analysis, this difference is significant only for the gluinos [3].

where Δ is given by,

$$\Delta = \left(\frac{3}{14}\right) \log \frac{m_S}{M_Z} + \left(\frac{8}{7}\right) \log \frac{M}{M_Z}. \tag{3.5}$$

Already we can see that raising the scale m_S or making gauginos and higgsinos heavier lowers the prediction for $\alpha_s(M_Z)$. Thus, split-SUSY can improve the agreement between $\alpha_s^{exp} = 0.119 \pm 0.002$ [11] and $\alpha_s^{MSSM,GUT}$ calculated assuming unification.

3.3 KK contributions and GUT thresholds

Near the compactification scale M_c, loops of KK modes lead to SU(5) universal power-law scaling of the couplings. There is also logarithmic non-universal running due to 4d brane kinetic terms and an effective zero mode mismatch that we will discuss shortly. As in section 2, we assume that M_c is sufficiently smaller than M_G to suppress brane kinetic contributions. Above M_c, the gauge couplings quickly become strong, so it is natural to assume that the unification scale coincides with strong coupling. If we have d extra dimensions, this assumption fixes the ratio $\frac{M_G}{M_c}$ as

$$\left(\frac{M_G}{M_c}\right)^d \approx \frac{16\pi^2}{Cg^2}, \tag{3.6}$$

where g^2 is evaluated at the compactification scale and C is a group theory factor ($C = 5$ for SU(5) or $C = 8$ for SO(10)) [12].

The effects of SU(5) violating brane kinetic operators can be seen be integrating over the extra dimension at the unification scale. For a single extra dimension of size R, the effective 4-D gauge coupling is

$$\frac{1}{\alpha_i} \approx \frac{R}{\alpha_{5D}} + \frac{1}{\alpha_i'}, \tag{3.7}$$

where α_{5D} is the bulk 5-D gauge coupling, and α_i' is the brane gauge coupling. Using the strong coupling assumption of $\alpha_{5D} M_G \approx \alpha' \approx 4\pi$, the SU(5) violating brane contributions are suppressed by $\frac{1}{RM_G} = \frac{M_c}{M_G}$ relative to the SU(5) preserving bulk. For the model we study, $\frac{M_c}{M_G} \approx \frac{1}{200}$ so that SU(5) violating brane contributions are entirely negligible.

By naive dimensional analysis, M_G scale threshold corrections to the α_i^{-1} should naturally be of order $\approx \frac{1}{4\pi}$. Thus, we expect the precision of our prediction for $\alpha_s(M_Z)$ to be limited by $\approx \frac{\alpha_s(M_Z)^2}{4\pi}\sqrt{3} \approx 0.002$. In addition, the effects of strong coupling over a small energy interval near M_G can be expected to make contributions of threshold size. In all, we estimate the uncertainty from GUT thresholds in our final calculation of α_s to be $\approx \pm 0.003$.

Assuming unification at M_G, the matching condition between the full theory and the 4d theory below M_c is,

$$\frac{1}{\alpha_i(M_c')} = \frac{1}{\alpha_G} + \frac{C}{4\pi}\left[\frac{M_G}{M_c} - 1\right] + \frac{\tilde{b}_i}{2\pi} \log \frac{M_G}{M_c'} + \frac{\Delta_i^{KK,thr}}{2\pi}, \tag{3.8}$$

where M_c' is the appropriate matching scale ($M_c' = \frac{M_c}{\pi}$ in the model of section 2), C reflects the contributions from the tower of KK modes that lead to universal power-law scaling,
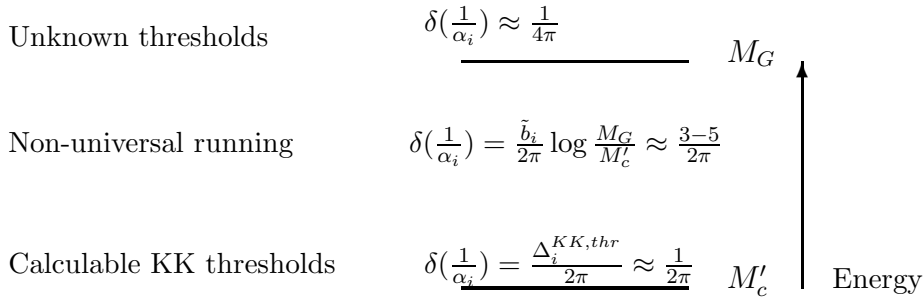
Unknown thresholds	$\delta(\frac{1}{\alpha_i}) \approx \frac{1}{4\pi}$	M_G
Non-universal running	$\delta(\frac{1}{\alpha_i}) = \frac{\tilde{b}_i}{2\pi} \log \frac{M_G}{M_c'} \approx \frac{3-5}{2\pi}$	
Calculable KK thresholds	$\delta(\frac{1}{\alpha_i}) = \frac{\Delta_i^{KK,thr}}{2\pi} \approx \frac{1}{2\pi}$	M_c' Energy

Figure 2: Summary of threshold corrections and non-universal running between M_c' and M_G.

$\tilde{b}_i$ are non-universal β-function coefficients, and $\Delta_i^{KK,thr}$ are threshold contributions. The thresholds $\Delta_i^{KK,thr}$ come from integrating out the gauge and matter KK modes and can be calculated given a choice for the matter representation and bulk geography. The bulk SU(5)-universal scaling controlled by C can be calculated given a suitable UV completion, but we will not bother with this because unification is not altered by this scaling. We instead use the estimate $C \approx 5$ for SU(5) and $C \approx 8$ for SO(10) [12].

In our analysis, the contributions from non-universal running are absorbed into a matching condition between α_3, α_2, and α_1 at the scale M_c'. Again taking the linear combination $\alpha_3^{-1} - \frac{12}{7}\alpha_2^{-1} + \frac{5}{7}\alpha_1^{-1}$, we obtain

$$\alpha_s^{-1}(M_c') = \frac{12}{7}\alpha_2^{-1}(M_c') - \frac{5}{7}\alpha_1^{-1}(M_c') + \frac{\Delta_{KK}}{2\pi}, \tag{3.9}$$

where Δ_{KK} is

$$\Delta_{KK} = \left(\tilde{b}_3 - \frac{12}{7}\tilde{b}_2 + \frac{5}{7}\tilde{b}_1\right) \log \frac{M_G}{M_c'} + \Delta_{KK}^{thr}, \tag{3.10}$$

and $\Delta_{KK}^{thr} = (\Delta_3^{KK,thr} - \frac{12}{7}\Delta_2^{KK,thr} + \frac{5}{7}\Delta_1^{KK,thr})$. From this relation, we obtain an additional contribution to $\delta\alpha_s(M_Z)$ of,

$$\delta\alpha_s(M_Z) \approx -\frac{\alpha_s(M_Z)^2}{2\pi}\Delta_{KK}. \tag{3.11}$$

So, for $\Delta_{KK} \geq 0$, the prediction for $\alpha_s(M_Z)$ is lowered, while for $\Delta_{KK} \leq 0$ it is increased. This effect, and the competing effect in eq. (3.4) will determine the favored location for the SM higgs fields in our toy model as well as the preferred range of SUSY breaking mass scales.

The simplest way to calculate the non-universal β-function coefficients $\tilde{b}_i$ is to think about what the boundary conditions do to the spectrum of the bulk theory [12]. For example, consider the simple case where the extra dimension is a circle S^1 orbifolded with a discrete Z_2 symmetry, $y \to -y$. The boundary conditions on the bulk fields at the fixed point of Z_2 result from their Z_2 parities and divide the KK tower into states that are even and odd under Z_2. The full KK tower of states on S^1 has completely SU(5) invariant running because there are no SU(5) violating defects in the bulk. Under the orbifold map, $S^1 \to \frac{S^1}{Z_2}$, half of the states are projected out (i.e. left and right moving states are mapped

onto single states). For a particular bulk state T with β-function coefficients b_i, let T_n^O ($n \geq 0$) and T_n^E ($n \geq 0$) be the odd and even modes with masses $m_n^O = \frac{n+1/2}{R}$ and $m_n^E = \frac{n}{R}$ respectively. The contribution from the odd states T_n^O is therefore equivalent to a tower of states $\tilde{T}_n^O$ ($\infty \geq n \geq -\infty$) on S^1 with beta function coefficients $b_i/2$. For the even states, we can do the same except that the zero mode will not in general have a β-function coefficient $b_i/2$. So, for the even modes, we can construct a tower of states equivalent to an S^1 tower with an effective zero mode with coefficient $b_i^0 - b_i/2$, where b_i^0 is the actual zero mode β-function coefficient. It is precisely this effective zero mode that generates the one-loop non-universal running.

This analysis can be easily generalized. If a manifold F is orbifolded under M such that, apart from the zero mode, n_M states of F are mapped to $\frac{F}{M}$, then the $\tilde{b}_i$ are given by,

$$\tilde{b}_i = b_i^0 - \frac{b_i^{KK}}{n_M}, \tag{3.12}$$

where b_i^0 is the zero mode β-function coefficient and b_i^{KK} is the the β-function coefficient of the excited KK states in the tower connected to the zero mode. For the parity assignments of the model in section 2, the KK tower connected to the zero modes has states with the same quantum numbers as the zero modes but with N=1 5d SUSY (i.e. N=2 4d SUSY). For example, suppose the zero mode fills out a N=1 SU(m) vector field V coming from a 5d N=1 vector $\{V, \Sigma\}$. The KK modes connected to the zero mode will then have a one-loop β-function coefficient, $b_{SU(m)}^{KK} = -2m$. On the other hand, the zero mode β-function coefficient is, $b_{SU(m)}^0 = -3m$. So, $b_i^{KK} = \frac{2}{3}b_i^0$ and $\tilde{b}_i = \frac{2}{3}b_i^0$. Generally, $\tilde{b}_i = \frac{2}{3}b_i^0$ for a N=1 SUSY vector V zero mode that comes from a bulk 5d SUSY vector in the adjoint. Analogous calculations show that, $b_i^{KK} = -2b_i^0$, and $b_i^{KK} = +2b_i^0$ when the zero mode is an adjoint Σ, or a fundamental Φ respectively. Thus, with $n_M = n$, we have $\tilde{b}_i = (1 - \frac{2}{3n})b_i^0$, $\tilde{b}_i = (1 + \frac{2}{n})b_i^0$, and $\tilde{b}_i = (1 - \frac{2}{n})b_i^0$ when the zero mode is a V, Σ, or Φ respectively.

With the above result, we can explicitly calculate the $\tilde{b}_i$ in our case. First, the gauge fields come from a bulk vector field, and the zero mode β-function coefficients are given by $(b_1^0, b_2^0, b_3^0) = (0, -6, -9)$ for the MSSM. Thus, $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (0, -6 + \frac{4}{n}, -9 + \frac{6}{n})$ for the gauge fields and so $\tilde{b}_{gauge} = \frac{1}{7}(9 - \frac{6}{n})$. The zero mode matter fields all come in complete SU(5) multiplets, so they do not contribute to non-universal running even if they come from bulk fields. As for the Higgs fields, they can come from brane fields, bulk hypermultiplet fields, or bulk gauge fields with the result that $\tilde{b}_{higgs,brane} = -\frac{9}{7}$, $\tilde{b}_{higgs,hyper} = -\frac{9}{7} + \frac{18}{7n}$, or $\tilde{b}_{higgs,gauge} = -\frac{9}{7} - \frac{18}{7n}$ respectively. In all, we therefore have,

$$\Delta_{KK} = \left(\frac{-6}{7n}\right) \log \frac{M_G}{M_c'} \quad \text{(brane localized Higgs)},$$

$$\Delta_{KK} = \left(\frac{12}{7n}\right) \log \frac{M_G}{M_c'} \quad \text{(Higgs from bulk hypermultiplets)},$$

$$\Delta_{KK} = \left(\frac{-24}{7n}\right) \log \frac{M_G}{M_c'} \quad \text{(Higgs from bulk vector multiplets)}. \tag{3.13}$$

In the above, we've neglected $\Delta_i^{KK,thr}$ contributions, but we included them in our final two-loop analysis. For reference, $\Delta_{KK}^{thr} = 0.84$ for the brane and bulk hypermultiplet Higgs cases, and $\Delta_{KK}^{thr} = -1.68$ for the bulk vector multiplet Higgs case [20].

Recalling that $\Delta_{KK} \geq 0$ lowers the prediction for $\alpha_s(M_Z)$, we see that Higgs fields coming from bulk hypermultiplets lower $\alpha_s(M_Z)$ while brane and bulk vector multiplet Higgs fields increase $\alpha_s(M_Z)$.

4. Results for gauge coupling unification

In the spirit of the one-loop analysis above, we performed a two-loop analysis using the model of section 2 to quantify what the unification predictions for α_s teach us about the bulk "geography" of the theory in light of the split-SUSY scenario. In particular, we investigated the preferred scale of scalar superpartner masses in split-SUSY and the preferred location of the Higgs fields in extra dimensional GUTs illustrated by this model. In order to be as model independent as possible, we assumed complete mass degeneracy of the squarks, sleptons, and charged and pseudoscalar Higgs, and neglected one-loop thresholds at m_S. We included the yukawa contributions of only the third generation of SM quarks and leptons in the two-loop RG running of the gauge couplings above M_Z. We also assumed higgsino and gaugino mass unification at M_c'.

For every choice of m_S and M, M_c and $\frac{M_c}{M_G}$ were calculated so that α_1 and α_2 unify and are both equal to 4π at M_G. Because varying the SUSY breaking scale m_S in Split-SUSY does not change the ordinary 4-D unified coupling α_G appreciably, the compactification scale M_c does not vary much as m_S is changed and is typically $M_c \approx 10^{15}$ GeV. For the 5-D SU(5) model illustrated above, we took $n_M = 2$, $d = 1$, and $C = 5$ to calculate the logarithmic corrections to the gauge coupling matching conditions at M_c'. For our analysis of 6-D SO(10) cases, we took $n_M = 2$, $d = 2$, and $C = 8$. Thus, we ignored any additional model dependent details for 6-D cases. Our results are given for higgsino and gaugino unified mass boundary conditions of 300 GeV and 1000 GeV and m_S is varied from 10^5 to 10^{14} GeV.

Figure 3, shows our two-loop results for unification in the original 4d split-SUSY model of [2–4]. The behavior of $\alpha_s(M_Z)$ as a function of m_S and M is clearly well described by eqs. (3.4) and (3.5) and is in agreement with [3].

Figure 4 shows our results for bulk hypermultiplet Higgs fields in the 5d SU(5) model of section 2 and a 6d SO(10) model. As expected from eqs. (3.11) and (3.13), we can see that the effects of non-universal GUT scale running and increasing m_S both lower $\alpha_s(M_Z)$ and consequently disfavor split-SUSY with bulk hypermultiplet Higgs fields. The magnitude of the KK contributions scale like $\frac{1}{d}$, and so the 6d model is disfavored slightly less. In both cases, $m_S \leq 10^5$ GeV and light gauginos and higgsinos are required in order to match experiment. c Figure 5 displays our unification results for brane Higgs fields in the 5d SU(5) model and 6d SO(10) model. In these cases, the effect of lowering $\alpha_s(M_Z)$ as m_S is increased is compensated by the positive contribution to $\alpha_s(M_Z)$ coming from non-universal running above M_c'. For the 5d SU(5) model, m_S is favored to be in the range $m_S = 10^{10\pm2}$ GeV, and for 6d SO(10) in the range $m_S = 10^{9\pm2}$ GeV.
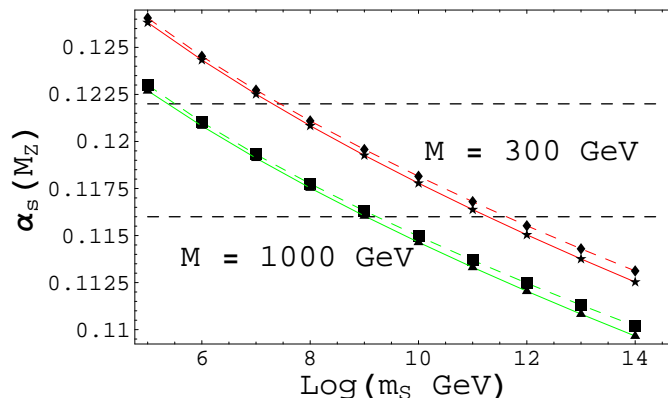
Figure 3: The prediction for $\alpha_s(M_Z)$ as a function of the SUSY breaking scale m_S for the 4D MSSM. The horizontal dashed lines show the 1σ experimental constraint for $\alpha_s^{exp}(M_Z)$ [11]. The solid lines correspond to $\tan(\beta) = 50$ and the dashed lines to $\tan(\beta) = 1.5$. We assume higgsino and gaugino mass unification at the unification scale.
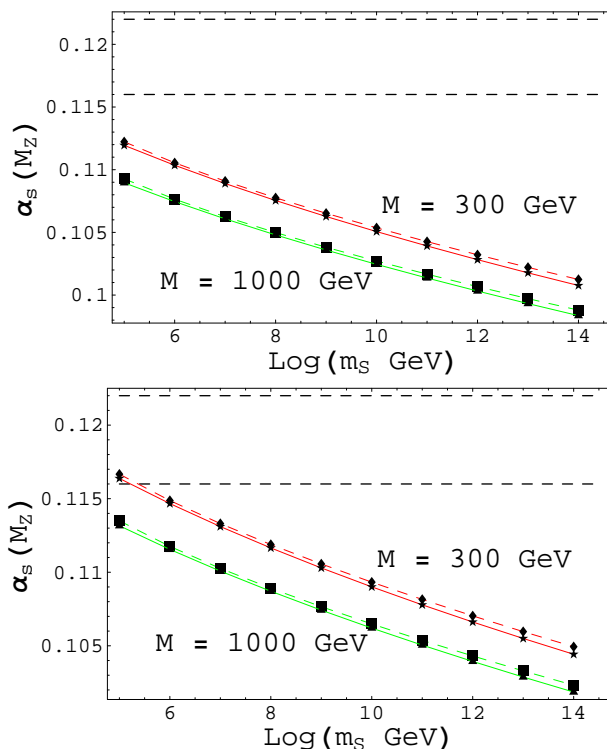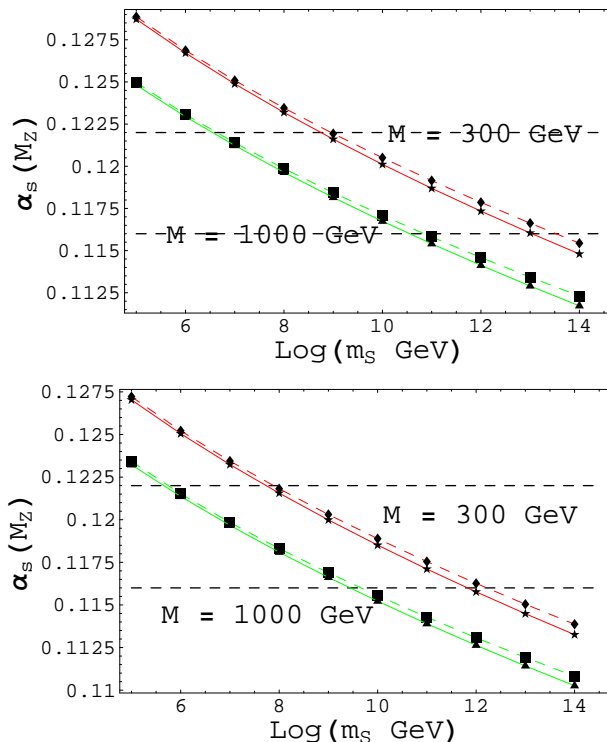

Figure 4: The prediction for $\alpha_s(M_Z)$ as a function of the SUSY breaking scale m_S. The top graphic is for the 5D SU(5) model with bulk hypermultiplet Higgs fields and the bottom graphic is for a 6D SO(10) model with bulk hypermultiplet Higgs fields. The horizontal dashed lines show the 1σ experimental constraint for $\alpha_s^{exp}(M_Z)$ [11]. The solid lines correspond to $\tan(\beta) = 50$ and the dashed lines to $\tan(\beta) = 1.5$. The compactification scale is set to $M_c' = 4 \times 10^{14}\,\mathrm{GeV}$. We assume higgsino and gaugino mass unification with mass M at the compactification scale. Both $M = 300\,\mathrm{GeV}$ and $M = 1000\,\mathrm{GeV}$ are shown.

Figure 5: The prediction for $\alpha_s(M_Z)$ as a function of the SUSY breaking scale m_S. The top graphic is for the 5D SU(5) model with brane Higgs fields and the bottom graphic is for a 6D SO(10) model with brane Higgs fields. The horizontal dashed lines show the 1σ experimental constraint for $\alpha_s^{exp}(M_Z)$ [11]. The solid lines correspond to $\tan(\beta) = 50$ and the dashed lines to $\tan(\beta) = 1.5$. The compactification scale is set to $M_c' = 4 \times 10^{14}\,\text{GeV}$. We assume higgsino and gaugino mass unification with mass M at the compactification scale. Both $M = 300\,\text{GeV}$ and $M = 1000\,\text{GeV}$ are shown.

According to eq. (3.13), the same counter competing effects on $\alpha_s(M_Z)$ occur for the bulk vector multiplet Higgs case as well. However, the magnitude of the non-universal contribution is four times as large than in the brane Higgs case and so all but very large m_S is disfavored as is shown in figure 6. This problem is less severe for the 6d SO(10) model where the bulk vector multiplet Higgs case requires at least $m_S = 10^{13}\,\text{GeV}$ and heavy gauginos and higgsinos to not be disfavored. So in this case, the preferred SUSY breaking scale coincides with the compactification scale.

5. Conclusions

The split-SUSY scenario offers an interesting new framework for beyond the standard model physics. Not motivated by naturalness as in the MSSM, unification becomes one of the central motivations for this scenario. Higher dimensional orbifold GUTs offer a particularly compelling unification framework in which many of the standard problems of SUSY GUTs can be overcome. In this spirit, we incorporated split-SUSY into a 5d SU(5) orbifold GUT. The primary constraint on these models is a successful prediction
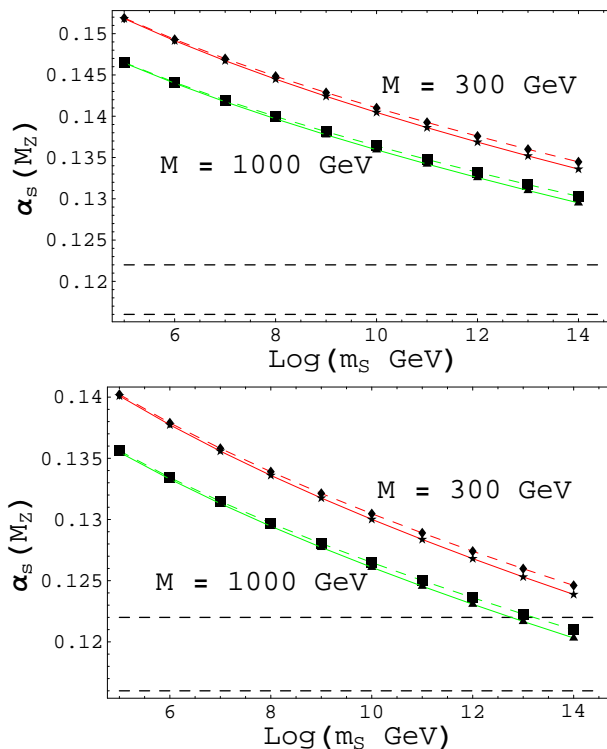
Figure 6: The prediction for $\alpha_s(M_Z)$ as a function of the SUSY breaking scale m_S. The top graphic is for the 5D SU(5) model with the Higgs coming from a bulk vector and the bottom graphic is for a 6D SO(10) model with the Higgs coming from a bulk vector. The horizontal dashed lines show the 1σ experimental constraint for $\alpha_s^{exp}(M_Z)$ [11]. The solid lines correspond to $\tan(\beta) = 50$ and the dashed lines to $\tan(\beta) = 1.5$. The compactification scale is set to $M_c' = 4 \times 10^{14}\,\mathrm{GeV}$. We assume higgsino and gaugino mass unification with mass M at the compactification scale. Both $M = 300\,\mathrm{GeV}$ and $M = 1000\,\mathrm{GeV}$ are shown.

of low energy gauge couplings. In generic orbifold GUT models, non-universal running above the compactification scale alters the low energy unification prediction of $\alpha_s(M_Z)$. The magnitude and sign of these contributions depends primarily on the bulk geography of the Higgs fields. On the other hand, lifting the scale of supersymmetry breaking lowers the unification prediction for $\alpha_s(M_Z)$. Our one- and two-loop analysis of gauge coupling unification shows that split-SUSY favors brane Higgs fields and relatively high scales of SUSY breaking of order $10^{10\pm2}\,\mathrm{GeV}$.

Acknowledgments

References

[1] S. Dimopoulos and H. Georgi, *Softly broken supersymmetry and* SU(5), *Nucl. Phys.* **B 193** (1981) 150;
N. Sakai, *Naturalness in supersymmetric 'guts'*, *Z. Physik* **C 11** (1981) 153;
S. Dimopoulos, S. Raby and F. Wilczek, *Supersymmetry and the scale of unification*, *Phys. Rev.* **D 24** (1981) 1681;
L.E. Ibanez and G.G. Ross, *Low-energy predictions in supersymmetric grand unified theories*, *Phys. Lett.* **B 105** (1981) 439.

[2] N. Arkani-Hamed and S. Dimopoulos, *Supersymmetric unification without low energy supersymmetry and signatures for fine-tuning at the LHC*, *JHEP* **06** (2005) 073 [hep-th/0405159].

[3] G.F. Giudice and A. Romanino, *Split supersymmetry*, *Nucl. Phys.* **B 699** (2004) 65 [hep-ph/0406088].

[4] N. Arkani-Hamed, S. Dimopoulos, G.F. Giudice and A. Romanino, *Aspects of split supersymmetry*, *Nucl. Phys.* **B 709** (2005) 3 [hep-ph/0409232].

[5] Y. Kawamura, *Gauge symmetry reduction from the extra space* S(1)/Z(2), *Prog. Theor. Phys.* **103** (2000) 613 [hep-ph/9902423].

[6] Y. Kawamura, *Triplet-doublet splitting, proton stability and extra dimension*, *Prog. Theor. Phys.* **105** (2001) 999 [hep-ph/0012125].

[7] Y. Kawamura, *Split multiplets, coupling unification and extra dimension*, *Prog. Theor. Phys.* **105** (2001) 691 [hep-ph/0012352].

[8] G. Altarelli and F. Feruglio, SU(5) *grand unification in extra dimensions and proton decay*, *Phys. Lett.* **B 511** (2001) 257 [hep-ph/0102301].

[9] L.J. Hall and Y. Nomura, *Gauge unification in higher dimensions*, *Phys. Rev.* **D 64** (2001) 055003 [hep-ph/0103125].

[10] P. Langacker and N. Polonsky, *The strong coupling, unification and recent data*, *Phys. Rev.* **D 52** (1995) 3081 [hep-ph/9503214];
J. Bagger, K.T. Matchev and D. Pierce, *Precision corrections to supersymmetric unification*, *Phys. Lett.* **B 348** (1995) 443 [hep-ph/9501277].

[11] PARTICLE DATA GROUP collaboration, *Review of particle physics. Particle data group*, S. Eidelman et al., *Phys. Lett.* **B 592** (2004) 1.

[12] L.J. Hall and Y. Nomura, *Gauge coupling unification from unified theories in higher dimensions*, *Phys. Rev.* **D 65** (2002) 125012 [hep-ph/0111068].

[13] L.J. Hall and Y. Nomura, *A complete theory of grand unification in five dimensions*, *Phys. Rev.* **D 66** (2002) 075004 [hep-ph/0205067].

[14] L.J. Hall and Y. Nomura, *Grand unification in higher dimensions*, *Ann. Phys. (NY)* **306** (2003) 132 [hep-ph/0212134].

[15] A. Hebecker and J. March-Russell, *A minimal* S(1)/(Z(2)xZ′(2)) *orbifold GUT*, *Nucl. Phys.* **B 613** (2001) 3 [hep-ph/0106166].

[16] L.J. Hall, Y. Nomura and D.R. Smith, *Gauge-higgs unification in higher dimensions*, *Nucl. Phys.* **B 639** (2002) 307 [hep-ph/0107331].

[17] L.J. Hall, Y. Nomura, T. Okui and D.R. Smith, *SO(10) unified theories in six dimensions*, *Phys. Rev.* **D 65** (2002) 035008 [`hep-ph/0108071`].

[18] L.J. Hall, *Grand unification of effective gauge theories*, *Nucl. Phys.* **B 178** (1981) 75.

[19] H. Arason, D.J. Castano, B. Keszthelyi, S. Mikaelian, E.J. Piard, P. Ramond and B.D. Wright, *Renormalization group study of the standard model and its extensions. I. The standard model*, *Phys. Rev.* **D 46** (1992) 3945.

[20] R. Contino, L. Pilo, R. Rattazzi and E. Trincherini, *Running and matching from 5 to 4 dimensions*, *Nucl. Phys.* **B 622** (2002) 227 [`hep-ph/0108102`].